Braess Paradox in Electrical Networks – When more might mean less

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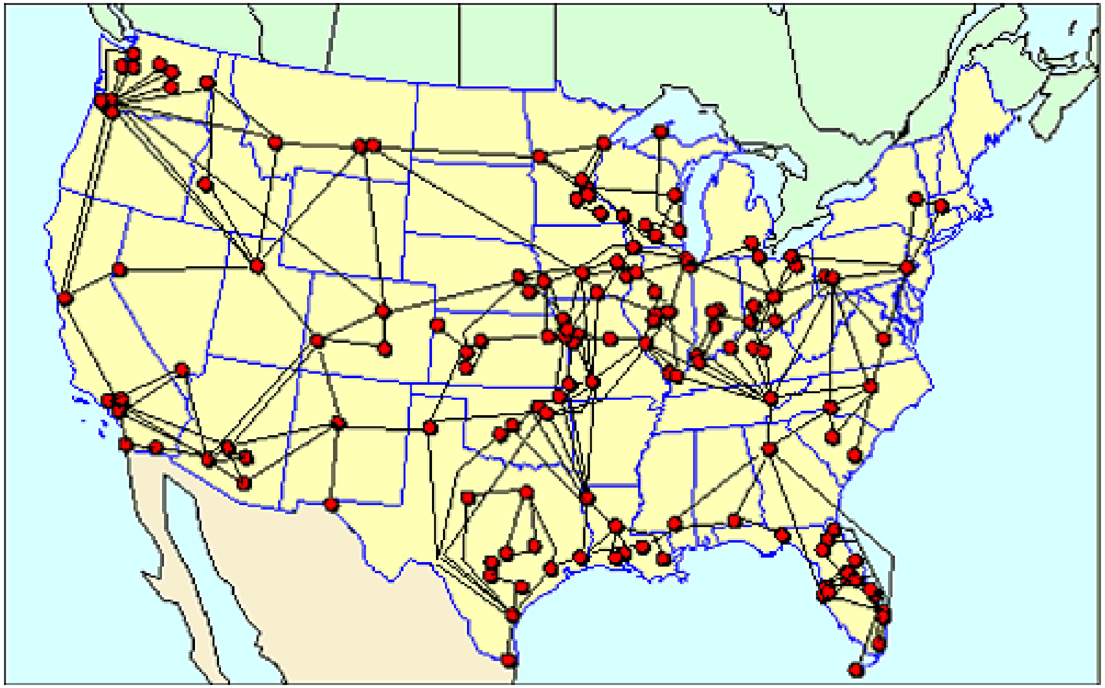
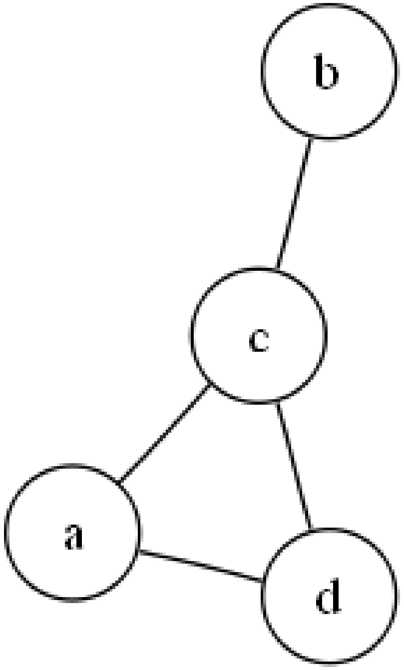


Motivations

Models of Sustainable Development using Smart Cities paradigm

- To develop an infrastructure capable to use resources in a programmed fashion by integrating Information Technologies into the spheres of economics, health, power generation, services, education, science, and politics.
- To optimize the power grid configuration (topology) via selecting the appropriate number of connections (hubs), loads, and wiring level (edges). Minimize the overloads and grid congestions.

Research Questions



Questions of Interest

1. How far the number of electrical lines determine the performance of a power grid?
2. How the performance of a power grid network can be assessed from its connectivity pattern?

Hypothesis 1: There is a critical number of lines above which the addition of an extra line will be detrimental rather than favorable for a better electrical flow. This hypothesis relates with the **Braess Paradox** in **Network Theory**.
Hypothesis 2: Power grid performance might be assessed through a combination of indices characterizing networks and enabling quantify the easiness of connecting two distant points by walking the shortest path.

Objectives

- Learn how to map electrical grids into mathematical networks and characterize them for structural and functional optimization.
- Learn about Braess Paradox and networks with Wheatstone Bridge configurations.
- Observe the presence of Braess Paradox in Wheatstone networks due to unmeet conditions.
- Developing an algorithm for detecting embedded Wheatstone sub-networks.
- Observe how extra Wheatstone connections in the network makes the system unbalanced, thus causing congestion in the flow across the network

Model Implementation

1. Map the electrical grid of USA into a theoretical network model and compute the associated **Adjacency Matrix** N_{ij} representing the connectivity between consecutive nodes of the power grid.
2. Compute the **power 2 and 3** of the **Adjacency Matrix** in order to assess the number of ways nodes might be accessed by making two or three walks.
3. Compute the **number of triangular configurations** among the number of nodes. Such configurations will be associated with the specificity of Wheatstone-bridge wiring of the network.
4. Compute the **geodesic path** between the elements of the network. It indicates the Min. distance across the network when nodes i and j are analyzed.
5. Compute the the Min. distance between Wheatstone-bridge patches in order to know points of electrical compensation of loads.

Wolfram Mathematica 10

N

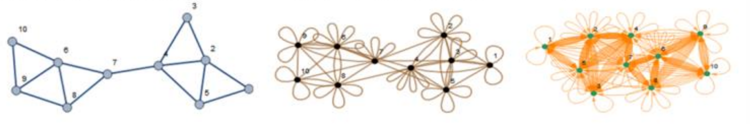
$$\begin{pmatrix} 0&1&0&0&1&0&0&0&0&0\\ 1&0&1&1&1&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0\\ 0&1&1&0&1&0&1&0&0&0\\ 1&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&1&1\\ 0&0&0&1&0&1&0&1&0&0\\ 0&0&0&0&0&1&1&0&1&0\\ 0&0&0&0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0&0&1&0 \end{pmatrix}$$

N^2

$$\begin{pmatrix} 2&1&1&2&1&0&0&0&0&0\\ 1&4&1&2&2&0&1&0&0&0\\ 1&1&2&1&2&0&1&0&0&0\\ 2&2&1&4&1&1&0&1&0&0\\ 1&2&2&1&3&0&1&0&0&0\\ 0&0&0&1&0&4&1&2&2&1\\ 0&1&1&0&1&1&3&1&2&1\\ 0&0&0&1&0&2&1&3&1&2\\ 0&0&0&0&0&2&2&1&3&1\\ 0&0&0&0&0&1&1&2&1&2 \end{pmatrix}$$

N^3

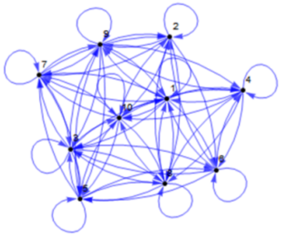
$$\begin{pmatrix} 2&6&3&3&5&0&2&0&0&0\\ 6&6&6&8&7&1&2&1&0&0\\ 3&6&2&6&3&1&1&1&0&0\\ 3&8&6&4&8&1&6&1&2&1\\ 5&7&3&8&4&1&1&1&0&0\\ 0&1&1&1&1&6&7&7&7&6\\ 2&2&1&6&1&7&2&6&3&3\\ 0&1&1&1&1&7&6&4&7&3\\ 0&0&0&2&0&7&3&7&4&5\\ 0&0&0&1&0&6&3&3&5&2 \end{pmatrix}$$



(a.)

$$\begin{pmatrix} 1&0&1&1&0&1&1&1&1&1\\ 0&1&0&0&0&1&1&1&1&1\\ 1&0&1&0&1&1&1&1&1&1\\ 1&0&0&1&0&1&0&1&1&1\\ 0&0&1&0&1&1&1&1&1&1\\ 1&1&1&1&1&1&0&0&0&0\\ 1&1&1&0&1&0&1&0&1&1\\ 1&1&1&1&1&0&0&1&0&1\\ 1&1&1&1&1&0&1&0&1&0\\ 1&1&1&1&1&0&1&1&0&1 \end{pmatrix}$$

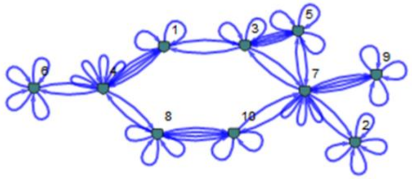
(b.)



(a.)

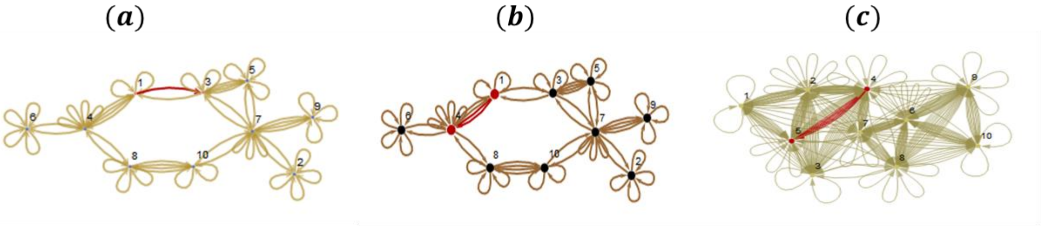
$$\begin{pmatrix} 2&0&1&2&0&0&0&0&0&0\\ 0&4&0&0&0&0&1&0&0&0\\ 0&0&2&0&2&0&1&0&0&0\\ 0&0&0&4&0&1&0&1&0&0\\ 0&0&0&0&3&0&1&0&0&0\\ 0&0&0&0&0&4&0&0&0&0\\ 0&0&0&0&0&0&3&0&2&1\\ 0&0&0&0&0&0&0&3&0&2\\ 0&0&0&0&0&0&0&0&3&0\\ 0&0&0&0&0&0&0&0&0&2 \end{pmatrix}$$

(b.)



Model Implementation

6. (a) Defines R1 as the set of all pairs of nodes that have one or two geodesic paths of length equal to one. (b) Defines R2 as the set of all pairs of nodes that have exactly two geodesic paths of length equal to one. Defines R3 as the set of all pairs of nodes that are separated by at least two paths of length 1.



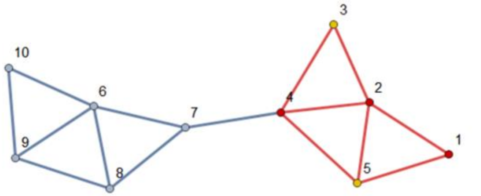
(a) {{1, 3}, {1, 4}, {2, 7}, {3, 5}, {3, 7}, {4, 6}, {4, 8}, {5, 7}, {7, 9}, {7, 10}, {8, 10}}

(b) {{1, 4}, {3, 5}, {7, 9}, {8, 10}}

(c) {{1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 7}, {2, 3}, {2, 4}, {2, 5}, {2,7}, {3, 4}, {3, 5}, {4, 5}, {4, 7}, {4, 9}, {6, 7}, {6, 8}, {6, 9}, {6, 10}, {7, 8}, {7, 9}, {7, 10}, {8, 9}, {8, 10}, {9, 10}}

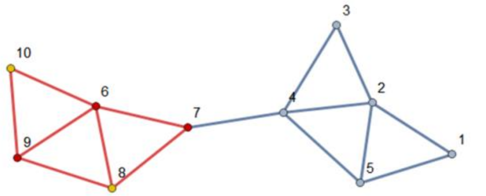
Wheatstone Bridge 1

	1	2	4	5
1	0	1	0	1
2	1	0	1	1
4	0	1	0	1
5	1	1	1	0



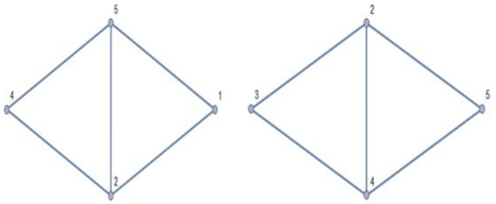
Wheatstone bridge 3

	6	8	9	10
6	0	1	1	1
8	1	0	1	0
9	1	1	0	1
10	1	0	1	0



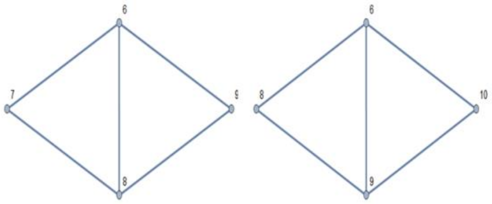
Wheatstone bridge 2

	2	3	4	5
2	0	1	1	1
3	1	0	1	0
4	1	1	0	1
5	1	0	1	0



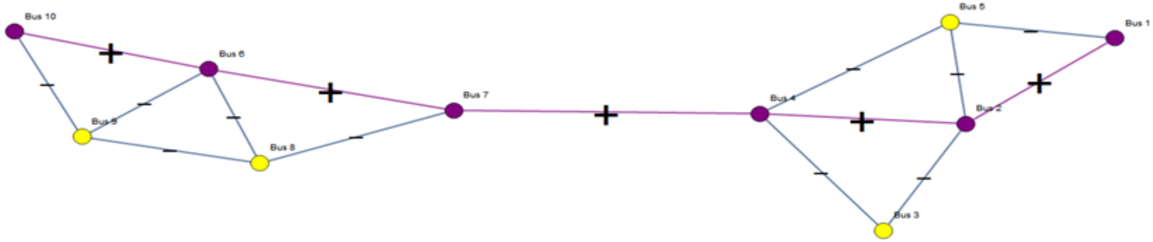
Wheatstone bridge 4

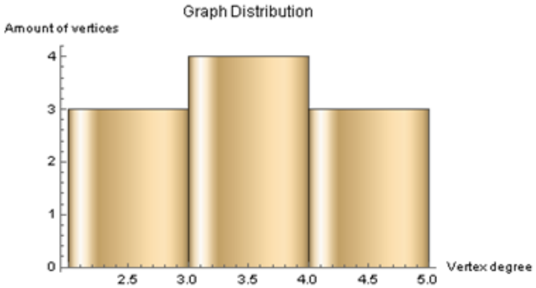
	6	7	8	9
6	0	1	1	1
7	1	0	1	0
8	1	1	0	1
9	1	0	1	0



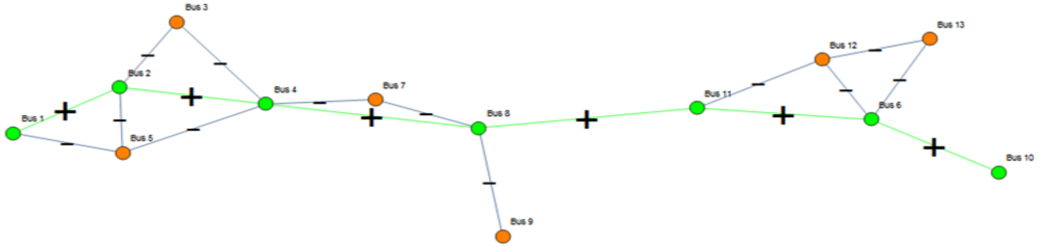
Demonstration of the Braess Paradox

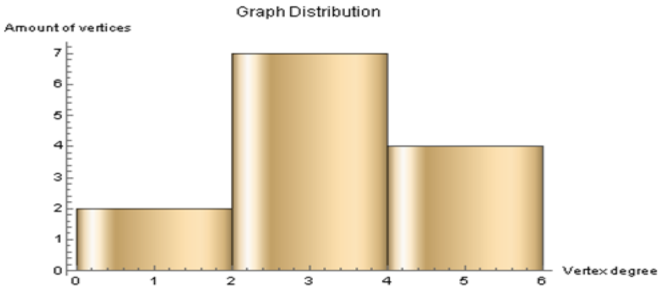
The reduced network graph where the highlighted lines represent the shortest and convenient path from one node to any other that traverses the least amount of resistances present . These are labeled with “+” while all others are indicated with a “-”.





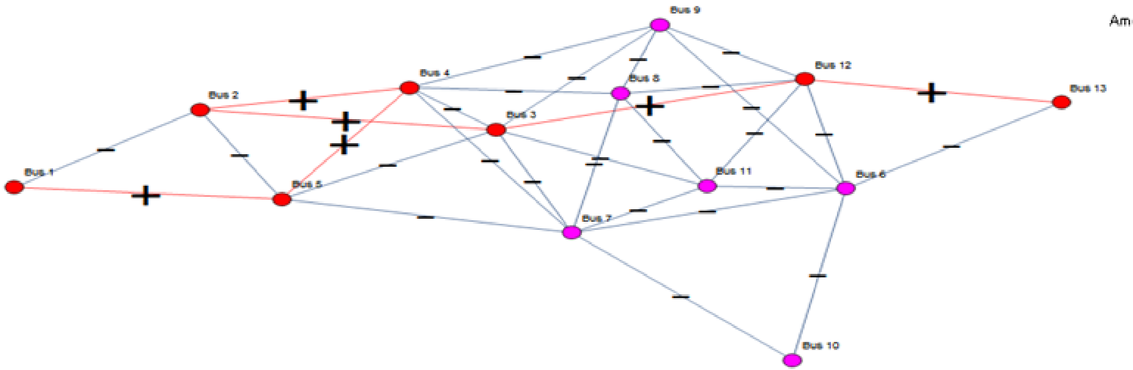
Upon creating the 13 - bus system with the added edges to the reduced network, we can see that based on the distribution of "+" & "-" lines in both network, the reduced network is balanced, but the 13 - bus network is unbalanced. Now, by comparing the values of the total resistance traversed between both networks, we see that the reduced network' s path was more efficient in avoiding the larger resistances.

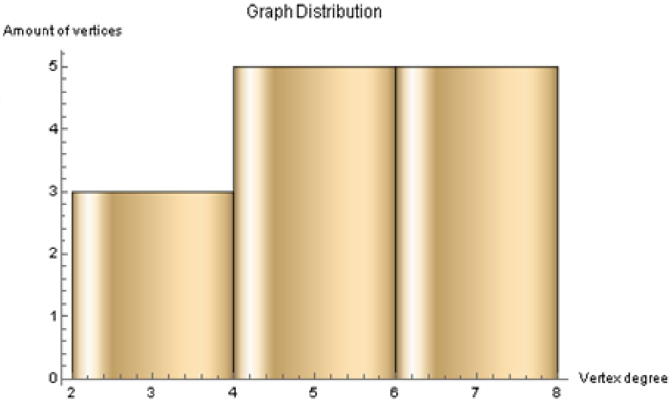




Demonstration of the Braess Paradox

To demonstrate how adding more edges to the network, would increase the amount of congestion. We created a 13-bus network with 31 connections. Actually, **the more edges you add to the network, the more unbalanced the system becomes**. Even if you increase the amount of buses, the more complex the network, the more unbalanced it will become





Conclusions and References

Conclusions

1. Power grids can be mapped into networks of vertices (hubs) and edges (connecting lines).
2. Power grid networks can be studied through the Adjacency Matrix, Geodesic Paths, and Clustering coefficient.
3. Electrical networks behave similar to Road networks. In both cases, the addition of extra links (roads, connecting lines) yields to overloads and a detriment in performance. This fact is known as the Braess Paradox.
4. For every network there is a critical value of edges above which, any addition does not introduce any improvement in performance.
5. Intelligent Dashboard can be implemented to control the performance of the power grid in parallel to the management windows.

References

- [1] Blumsack, S., 2006. “Network Topologies and Transmission Investment Under Electric-Industry Restructuring,” unpublished Ph.D. dissertation, Department of Engineering and Public Policy, Carnegie Mellon University. Available at www.andrew.cmu.edu/~sblumsac.
- [2] Blumsack S. and Ilic M., "The Braess Paradox in Electric Power Systems",http://www.personal.psu.edu/sab51/braess ´ paradox.pdf

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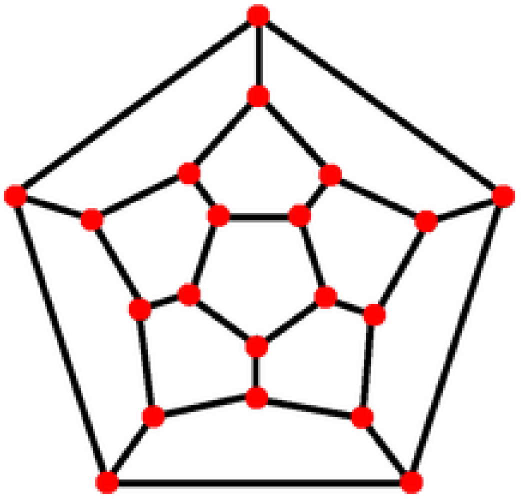

Connecting the World